Tracking controller design with preview action for a class of Lipschitz nonlinear systems and its applications

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Abstract: This paper addresses the problem of output tracking control with preview action for a class of Lipschitz nonlinear systems. By adopting a backward difference approach, an augmented error system (AES) is successfully constructed that fully utilizes the available future information in the reference signal as well as disturbances. The output tracking control problem with preview action is thereby reduced to a stabilization problem. Then, both the state feedback controller and the static output feedback controller are systematically developed. With the aid of the S-procedure and auxiliary matrix variable approach, sufficient design conditions for the asymptotic stability of the closed-loop system are presented through linear matrix inequality (LMI) formulation. Based on these criteria, two novel output tracking control laws are derived. Finally, two numerical examples are provided to highlight the effectiveness and superiority of the proposed control methodologies in terms of enhancing the overall tracking performance of the system.

Keywords: output tracking control; preview control; Lipschitz nonlinear systems; state feedback; static output feedback; LMI

1 Introduction

Output tracking control is a very important and hot research topic in the control community, and its importance is reflected by large coverage with numerous applications in linear motor control, flight control, fuel cells, digital communication networks and many other fields [1, 21, 22, 30, 37]. The main objective of output tracking control is to steer the output of a system, via a suitable controller, to follow the desired reference signal as closely as possible. Particularly, when some future information about the desired reference signal is known in advance, the overall tracking performance of closed-loop system can be largely improved through a preview control technique. This is mainly because that preview control fully incorporates available future knowledge for reference signals and/or disturbances into the controller structure implemented in a systematically reasonable manner. In recent decades, preview control has attracted tremendous attention and has been widely applied to output tracking control for various linear systems. Interesting works in this area were provided for linear discrete- or continuous-time systems [14, 15], linear discrete- or continuous-time descriptor systems [2, 19], linear discrete-time uncertain systems [17, 18], etc. Moreover, there have also been some reported results in the literature regarding preview tracking control for nonlinear systems. For instance, in [23], an optimal preview controller was proposed for nonlinear vehicle suspensions based on the nonlinear neutral network approach, resulting in distinct improvement of the response performance of the rear suspension. In [20], a direct control method was adopted to investigate the optimal preview control for discrete-time systems with specific nonlinear characteristics. In [35], a novel preview controller design procedure was developed by employing an information fusion technique, which was applicable to more general discrete-time nonlinear systems. It is necessary to note, however, that the preview control problem has not yet been adequately researched for nonlinear systems compared to linear counterparts.

The Lipschitz nonlinear system is one of the important classes of nonlinear systems and is composed of a linear part and a nonlinear part satisfying the Lipschitz condition. The famous single-link joint robot systems [34] and Chua’s circuit [12] can be represented in Lipschitz form. Because of its profound theoretical and engineering background, the Lipschitz nonlinear system has received considerable attention from many researchers, and extensive results have been reported. For example, in [41, 42, 44, 45], the design of different types of observers was well addressed for Lipschitz systems. The output feedback stabilization problem of Lipschitz nonlinear systems was considered in [25] by applying the H_∞ optimization principle. To address the unavailable states issue, an observer-based state feedback stabilization control scheme was proposed in [9]. Furthermore, in [26], an innovative nonlinear feedback control strategy was established for Lipschitz systems, which can achieve both stabilization and tracking control objectives. In [39, 40], a state feedback controller with integral action for tracking error was provided for Lipschitz nonlinear systems, resulting in asymptotic tracking of a constant reference signal. As a generalization of Lipschitz nonlinear systems, the robust H_∞ sliding mode control for discrete-time conic-type nonlinear systems was studied in [8]. Apart from the aforementioned studies, other control applications including synchronization control [3, 43, 46], adaptive control [36, 47], consensus control [7, 10] and finite-time control [27, 29] have been extensively investigated. However, few results concerning the preview control problem of general Lipschitz nonlinear systems can be found in the literature, which motivates this investigation.

The goal of this paper is to design tracking control schemes with preview action for general Lipschitz nonlinear systems. The key idea is to transform the preview tracking control problem into a stabilization problem by constructing a suitable augmented error system (AES). Then, with the aid of some special mathematical derivations combined with the S-procedure and auxiliary matrix variables approach, both the state feedback control and the static output feedback control are systematically taken into consideration. As the first of its kind, this paper provides novel LMI-based tracking control schemes with preview action for discrete-time Lipschitz systems. In contrast to the existing results [26, 39], the proposed design method greatly relaxes the requirements for system matrices, and the obtained tracking control structure includes a preview compensation mechanism related to future reference and disturbance in addition to the usual feedback control and integral control action. Finally, the simulation results are presented to substantiate the effectiveness and feasibility of the proposed tracking control strategy.

Notation Throughout this paper, R^n denotes the n-dimensional Euclidean space, and $R^{n\times m}$ is the set of all $n\times m$ real matrices. The notation $P>0$ ($P<0$) is used to define a symmetric positive-definite (negative-definite) matrix. For matrices P and Q, $P>Q$ stands for $P-Q>0$. The notation $diag(A,B)$ denotes the block diagonal matrix whose diagonal elements are matrices A and B. M^T denotes the transpose of the matrix M. M^{-T} denotes the inverse of the matrix M^T, namely, $(M^T)^{-1}$. $Sym\{A\}$ is used to represent $A+A^T$. The symbol "*" represents the transposed element in the symmetric positions. I_r represents the identity matrix of dimension r. $|*|$ denotes the absolute value of scalar variables. $\|*\|$ denotes the Euclidean norm of vectors.

2 Problem formulation

Consider the following discrete-time nonlinear system

$$\begin{cases} x(k+1)=Ax(k)+f(x(k))+Bu(k)+Dd(k), \\ y(k)=Cx(k), \end{cases} \tag{1}$$

where $x(k)\in R^n$ is the state vector, $u(k)\in R^m$ is the input vector, $d(k)\in R^q$ is the disturbance vector, and $y(k)\in R^p$ is the output vector. A,B,C,D are constant matrices with appropriate dimensions. $f(x)\in R^n$ is a nonlinear function vector.

Throughout the paper, the following assumptions are needed.

Assumption 1. The nonlinearity $f(x)$ is globally Lipschitz with respect to x, i.e., there exists a constant $\gamma>0$ such that for all $x,\bar{x}\in R^n$ the following holds

$$\|f(x)-f(\bar{x})\|\le\gamma\|x-\bar{x}\|. \tag{2}$$

Remark 1. Assumption 1 is referred to as a globally Lipschitz condition with a Lipschitz constant γ. The category of nonlinear systems satisfying Assumption 1 covers a large range of physical systems in the real world; among them, two typical examples are the robot model [25, 34] and chaotic Chua's circuit [12]. In current studies [11, 16, 38-40], the function $f(x)$ is required to vanish at the origin (i.e. $f(0)=0$). This restriction is not required in this paper by adopting the classical difference approach. Moreover, the nonlinearity $f(x)$ verifying this assumption can be nondifferentiable. Thus, the class of Lipschitz systems covered by this paper is more general.

Assumption 2 [31] The disturbance signal $d(k)$ converges to a constant vector d as $k \to \infty$, i.e., $\lim_{k\to\infty} d(k)=d$. Furthermore, $d(k)$ is assumed to be previewable, and the preview length is M_d, that is, at each time k, M_d future values $d(k+1),\cdots,d(k+M_d)$, as well as the present and past values of the disturbance signal are available. Additionally, the future values of the disturbance beyond the time $k+M_d$ are assumed not to change, namely,

$$d(k+i)=d(k+M_d), i=M_d+1, M_d+2,\cdots.$$

The desired reference signal is $r(k)$ and satisfies the following assumption:

Assumption 3 [2] The reference signal $r(k)$ converges to a constant vector r as $k \to \infty$, i.e., $\lim_{k\to\infty} r(k)=r$. Furthermore, $r(k)$ is assumed to be previewable, and the preview length is M_r, that is, at each time k, M_r future values $r(k+1),\cdots,r(k+M_r)$, as well as the present and past values of the reference signal are available. Additionally, the future values of the reference signal beyond the time $k+M_r$ are assumed not to change, namely,

$$r(k+i)=r(k+M_r), i=M_r+1, M_r+2,\cdots.$$

Remark 2. The previewable characteristics of the disturbance and reference signal are presented in Assumptions 2 and 3, which are rather standard hypotheses in the field of preview control. A large number of studies and experiments demonstrate that the output tracking performance of the closed-loop system can be largely improved by fully utilizing preview information in a control structure [2, 14, 15, 17-20, 23, 35]. As noted in [2, 31, 32], the future information on the reference signal and disturbance are less important as they exceed the preview range, thus, it is always assumed that the values beyond the preview range are constant.

The error signal is defined as follows:

$$e(k) = y(k) - r(k). \tag{3}$$

The objective of the paper is to design a suitable preview controller that makes the closed-loop output vector $y(k)$ track the reference signal $r(k)$ without static error, that is,

$$\lim_{k\to\infty} e(k) = \lim_{k\to\infty}\left(y(k) - r(k)\right) = 0.$$

The following lemmas are needed to derive the main results of this paper.

Lemma 1 (Schur complement lemma) [6] The symmetric matrix $\begin{bmatrix} S_{11} & S_{12} \\ S_{12}^T & S_{22} \end{bmatrix} < 0$ if and only if one of the following two conditions is satisfied:

(i) $S_{11} < 0, S_{22} - S_{12}^T S_{11}^{-1} S_{12} < 0$;

(ii) $S_{22} < 0, S_{11} - S_{12} S_{22}^{-1} S_{12}^T < 0$.

Lemma 2 [5] For matrices T, M, S, N with appropriate dimensions and a scalar β. The inequality

$$T + MS + S^T M^T < 0$$

is fulfilled if the following condition holds:

$$\begin{bmatrix} T & * \\ \beta M^T + NS & -\beta N - \beta N^T \end{bmatrix} < 0.$$

Lemma 3 [24] If there exist matrices $P > 0$ and G with appropriate dimensions, then

$$-G^T P^{-1} G \le P - G - G^T.$$

3 Construction of the AES

In preview control theory, the AES containing the tracking error and the preview information plays a significant role in the preview controller design. In this section, we adopt the classical difference method to derive the AES.

First, we introduce the first-order backward difference operator Δ:

$$\Delta x(k) = x(k) - x(k-1).$$

Applying the operator to both sides of (1) leads to

$$\Delta x(k+1) = A\Delta x(k) + \Delta f_k + B\Delta u(k) + D\Delta d(k), \tag{4}$$

where $\Delta f_k = \Delta f(x(k)) = f(x(k)) - f(x(k-1))$ is the difference of the nonlinearity. Then, applying the same operator on the error signal yields $\Delta e(k) = \Delta y(k) - \Delta r(k)$. Furthermore, from the output equation of (1) and (4), $\Delta e(k)$ is governed by

$$\Delta e\left(k+1\right) = CA\Delta x(k) + C\Delta f_k + CB\Delta u(k) + CD\Delta d(k) - \Delta r(k+1).$$

Since $\Delta e(k+1) = e(k+1) - e(k)$, the error dynamics are given by

$$e(k+1) = e(k) + CA\Delta x(k) + C\Delta f_k + CB\Delta u(k) + CD\Delta d(k) - \Delta r(k+1). \tag{5}$$

Putting (4) and (5) together results in

$$\tilde{x}(k+1) = \tilde{A}\tilde{x}(k) + \tilde{B}\Delta u(k) + G_r\Delta r(k+1) + \tilde{D}\Delta d(k) + G_f\Delta f_k, \tag{6}$$

where

$$\tilde{x}(k) = \begin{bmatrix} e(k) \\ \Delta x(k) \end{bmatrix}, \tilde{A} = \begin{bmatrix} I & CA \\ 0 & A \end{bmatrix}, \tilde{B} = \begin{bmatrix} CB \\ B \end{bmatrix}, G_r = \begin{bmatrix} -I_p \\ 0 \end{bmatrix}, G_d = \begin{bmatrix} CD \\ D \end{bmatrix}, G_f = \begin{bmatrix} C \\ I_n \end{bmatrix}.$$

To introduce the preview information on reference signal $r(k)$ and disturbance $d(k)$, we define new vectors:

$$x_r(k) = \begin{bmatrix} \Delta r(k) \\ \Delta r(k+1) \\ \vdots \\ \Delta r(k+M_r) \end{bmatrix} \in R^{p(M_r+1)}, \quad x_d(k) = \begin{bmatrix} \Delta d(k) \\ \Delta d(k+1) \\ \vdots \\ \Delta d(k+M_d) \end{bmatrix} \in R^{q(M_d+1)}.$$

From Assumptions 2 and 3, it is easily seen that

$$x_r(k+1) = A_r x_r(k), \tag{7}$$

$$x_d(k+1) = A_d x_d(k), \tag{8}$$

where

$$A_r=\left[\begin{array}{c:cccc}0 & I_p & 0 & \cdots & 0\\ 0 & 0 & I_p & \cdots & 0\\ \vdots & \vdots & \vdots & \ddots & \vdots\\ 0 & 0 & 0 & \cdots & I_p\\ \hdashline 0 & 0 & 0 & \cdots & 0\end{array}\right],\ A_d=\left[\begin{array}{c:cccc}0 & I_q & 0 & \cdots & 0\\ 0 & 0 & I_q & \cdots & 0\\ \vdots & \vdots & \vdots & \ddots & \vdots\\ 0 & 0 & 0 & \cdots & I_q\\ \hdashline 0 & 0 & 0 & \cdots & 0\end{array}\right].$$

It should be noted that the vectors $x_r(k)$ and $x_d(k)$ are given by using the preview, and all the future information about the reference signal and disturbance available at time k is summarized in (7) and (8). These two equations are crucial to the preview control system design.

Let $s \triangleq p+n+p(M_r+1)+q(M_d+1)$, define an augmented state vector $\bar{x}(k)=\begin{bmatrix}\tilde{x}^T(k) & x_r^T(k) & x_d^T(k)\end{bmatrix}^T \in R^s$. Considering (6)-(8), one can obtain

$$\bar{x}(k+1)=\bar{A}\bar{x}(k)+\bar{B}\Delta u(k)+F\Delta f_k\,, \tag{9}$$

where

$$\bar{A}=\begin{bmatrix}\tilde{A} & \tilde{G}_r & \tilde{G}_d\\ 0 & A_r & 0\\ 0 & 0 & A_d\end{bmatrix},\bar{B}=\begin{bmatrix}\tilde{B}\\ 0\\ 0\end{bmatrix},F=\begin{bmatrix}G_f\\ 0\\ 0\end{bmatrix},\tilde{G}_r=\begin{bmatrix}0 & G_r & \cdots & 0\end{bmatrix},\tilde{G}_d=\begin{bmatrix}G_d & 0 & \cdots & 0\end{bmatrix}.$$

In the theory of preview control, equation (9) is usually called an AES, which appears in a similar form as the original system (1). To facilitate the following design analysis, define matrix $F_1=\begin{bmatrix}0 & I_n & 0 & 0\end{bmatrix}$. Then the relationship between $\Delta x(k)$ and the augmented state $\bar{x}(k)$ is given by

$$\Delta x(k)=F_1\bar{x}(k)\,. \tag{10}$$

Relationship (10) and Assumption 1 help us overcome the difficulty of dealing with the difference of nonlinear function in (9), especially when analyzing the system stability using the Lyapunov function approach.

Based on the above procedures, the tracking control problem with the preview action of system (1) is reduced into the stabilization problem of system (9). Suppose that there exists a controller such that the closed-loop system of system (9) is asymptotically stable. Then, the tracking error $e(k)$, as a state component of $\bar{x}(k)$, converges to the vector zero as time k tends toward infinity, i.e., $\lim_{k\to\infty} e(k) = 0$. As a result, the desired output tracking control objective is achieved.

4 Design of the preview controller

4.1 State feedback case

In this subsection, we consider the following state feedback controller for system (9)

$$\Delta u(k) = K_X \bar{x}(k), \tag{11}$$

where K_X is the gain matrix to be determined. Then applying this controller to system (9), the following closed-loop system is obtained

$$\bar{x}(k+1) = (\bar{A} + \bar{B}K_X)\bar{x}(k) + F\Delta f_k. \tag{12}$$

The sufficient condition for the stability of system (12) is presented and proved in Theorem 1.

Theorem 1: Suppose that Assumptions 1-3 are satisfied. System (12) is asymptotically stable if there exist matrices $P > 0, G_1, G_2, R$ and scalar $\mu > 0$ such that

$$\begin{bmatrix} P - Sym\{G_1\} & * & * & * \\ 0 & \mu I_n - Sym\{G_2\} & * & * \\ \bar{A}G_1 + \bar{B}R & FG_2 & -P & * \\ \gamma F_1 G_1 & 0 & 0 & -\mu I_n \end{bmatrix} < 0. \tag{13}$$

Furthermore, the gain matrix K_X is given by $K_X = RG_1^{-1}$.

Proof: Consider the following Lyapunov function:

$$V(\bar{x}) = \bar{x}P^{-1}\bar{x}. \tag{14}$$

It is clear that the function V is positive-definite. Calculating the difference of $V(\bar{x})$ along system (12) yields

$$
\begin{aligned}
\Delta V\left(\bar{x}(k)\right) &= \bar{x}^T(k)P^{-1}\bar{x}(k) - \bar{x}^T(k-1)P^{-1}\bar{x}(k-1) \\
&= \left((\bar{A}+\bar{B}K_X)\bar{x}(k-1) + F\Delta f_{k-1}\right)^T P^{-1}\left((\bar{A}+\bar{B}K_X)\bar{x}(k-1) + F\Delta f_{k-1}\right) \\
&\quad - \bar{x}^T(k-1)P^{-1}\bar{x}(k-1) \\
&= \bar{x}^T(k-1)\left((\bar{A}+\bar{B}K_X)^T P^{-1}(\bar{A}+\bar{B}K_X) - P^{-1}\right)\bar{x}(k-1) \\
&\quad + 2\bar{x}^T(k-1)(\bar{A}+\bar{B}K_X)^T P^{-1}F\Delta f_{k-1} + \Delta f_{k-1}^T F^T P^{-1}F\Delta f_{k-1}.
\end{aligned} \tag{15}
$$

From Assumption 1, the following inequality can be obtained:

$$
\gamma^2 \Delta x^T(k-1)\Delta x(k-1) - \Delta f_{k-1}^T \Delta f_{k-1} \geq 0\,. \tag{16}
$$

From (10), (16) can be rewritten as

$$
\gamma^2 \bar{x}^T(k-1)F_1^T F_1 \bar{x}(k-1) - \Delta f_{k-1}^T \Delta f_{k-1} \geq 0\,. \tag{17}
$$

Then, for arbitrary constant $\mu > 0$, one can readily obtain from (15) and (17) that

$$
\begin{aligned}
\Delta V\left(\bar{x}(k)\right) &\leq \bar{x}^T(k-1)\left((\bar{A}+\bar{B}K_X)^T P^{-1}(\bar{A}+\bar{B}K_X) - P^{-1}\right)\bar{x}(k-1) \\
&\quad + 2\bar{x}^T(k-1)(\bar{A}+\bar{B}K_X)^T P^{-1}F\Delta f_{k-1} + \Delta f_{k-1}^T F^T P^{-1}F\Delta f_{k-1} \\
&\quad + \mu^{-1}\gamma^2 \bar{x}^T(k-1)F_1^T F_1 \bar{x}(k-1) - \mu^{-1}\Delta f_{k-1}^T \Delta f_{k-1} \\
&= \bar{x}^T(k-1)\left((\bar{A}+\bar{B}K_X)^T P^{-1}(\bar{A}+\bar{B}K_X) - P^{-1} + \mu^{-1}\gamma^2 F_1^T F_1\right)\bar{x}(k-1) \\
&\quad + 2\bar{x}^T(k-1)(\bar{A}+\bar{B}K_X)^T P^{-1}F\Delta f_{k-1} + \Delta f_{k-1}^T(F^T P^{-1}F - \mu^{-1}I_n)\Delta f_{k-1} \\
&= \left[\bar{x}^T(k-1) \quad \Delta f_{k-1}^T\right]\Omega\left[\bar{x}^T(k-1) \quad \Delta f_{k-1}^T\right]^T,
\end{aligned}
$$

where

$$
\Omega = \begin{bmatrix} (\bar{A}+\bar{B}K_X)^T P^{-1}(\bar{A}+\bar{B}K_X) - P^{-1} + \mu^{-1}\gamma^2 F_1^T F_1 & * \\ F^T P^{-1}(\bar{A}+\bar{B}K_X) & F^T P^{-1}F - \mu^{-1}I_n \end{bmatrix}.
$$

By applying the S-procedure [28], we can obtain that ΔV is negative-definite if $\Omega < 0$. According to the Lyapunov stability theorem, the closed-loop system (12) is asymptotically stable.

Now we prove that $\Omega < 0$ holds if condition (13) in Theorem 1 is satisfied.

Applying Lemma 3, the inequality (13) guarantees that

$$
\begin{bmatrix} -G_1^T P^{-1} G_1 & * & * & * \\ 0 & -G_2^T(\mu I_n)^{-1}G_2 & * & * \\ \bar{A}G_1 + \bar{B}R & FG_2 & -P & * \\ \gamma F_1 G_1 & 0 & 0 & -\mu I_n \end{bmatrix} < 0\,. \tag{18}
$$

Pre- and post-multiplying (18) by the invertible matrix $diag(G_1^{-T}, G_2^{-T}, I_s, I_n)$ and its transpose, respectively, and using $R = K_X G_1$, it follows that

$$\begin{bmatrix} -P^{-1} & * & * & * \\ 0 & -(\mu I_n)^{-1} & * & * \\ \bar{A}+\bar{B}K_X & F & -P & * \\ \gamma F_1 & 0 & 0 & -\mu I_n \end{bmatrix} < 0 . \tag{19}$$

Note that the invertibility of G_i $(i=1,2)$ follows from $G_i + G_i^T > P$ and $P > 0$. Since $diag(-P, -\mu I_n) < 0$, employing Lemma 1 to (19), one can conclude that $\Omega < 0$ holds. This completes the proof.

If LMI (13) is feasible, then the state feedback controller that ensures the asymptotic stability of the closed-loop system (12) is given by (11) with $K_X = RG_1^{-1}$. To simplify the expression and to clarify the controller structure, K_X is partitioned as

$$K_X = \begin{bmatrix} K_e & K_x & K_r & K_d \end{bmatrix}, \tag{20}$$

$$K_r = \begin{bmatrix} k_r(0) & k_r(1) & \cdots & k_r(M_r) \end{bmatrix}, \tag{21}$$

$$K_d = \begin{bmatrix} k_d(0) & k_d(1) & \cdots & k_d(M_d) \end{bmatrix}, \tag{22}$$

where $K_e \in R^{m\times p}, K_x \in R^{m\times n}, K_r \in R^{m\times p(M_r+1)}, K_d \in R^{m\times q(M_d+1)}$, $k_r(i) \in R^{m\times p}$ for $i = 0,1,\cdots,M_r$ and $k_d(i) \in R^{m\times q}$ for $i = 0,1,\cdots,M_d$.

Then, (11) can be further rewritten as

$$\Delta u(k) = K_e e(k) + K_x \Delta x(k) + \sum_{i=0}^{M_r} k_r(i)\Delta r(k+i) + \sum_{i=0}^{M_d} k_d(i)\Delta d(k+i) .$$

From the definition of $\Delta u(k)$, the control input $u(k)$ is computed by

$$\begin{aligned} u(k) &= \sum_{i=0}^{k} \Delta u(i) + u(-1) \\ &= K_e \sum_{i=0}^{k} e(i) + K_x x(k) + \sum_{j=0}^{M_r} k_r(j) r(k+j) + \sum_{j=0}^{M_d} k_d(j) d(k+j) \\ &\quad - \sum_{j=1}^{M_r} k_r(j) r(j-1) - \sum_{j=1}^{M_d} k_d(j) d(j-1). \end{aligned}$$

Here, we have used the assumption that $x(i)=0, u(i)=0, r(i)=0$ and $d(i)=0$ for any $i<0$.

Based on the above analysis, the first main result of this paper is obtained as follows.

Theorem 2: Suppose that Assumptions 1-3 are satisfied. If LMI (13) in Theorem 1 has a feasible solution, then the preview tracking controller of system (1) is

$$\begin{aligned} u(k) = K_e \sum_{i=0}^{k} e(i) + K_x x(k) + \sum_{j=0}^{M_r} k_r(j) r(k+j) + \sum_{j=0}^{M_d} k_d(j) d(k+j) \\ - \sum_{j=1}^{M_r} k_r(j) r(j-1) - \sum_{j=1}^{M_d} k_d(j) d(j-1), \end{aligned} \tag{23}$$

where the related gain matrices are determined by (20)-(22). Under this controller, the output vector $y(k)$ tracks the reference signal $r(k)$ without static error.

Remark 3: The state feedback-based controller $u(k)$ given by (23) consists of six terms. The first term represents the integral control action of the tracking error, the second term represents the state feedback control action, the third and the fourth terms are the preview compensation actions based on future references and disturbances, and the last two parts depend on the initial preview knowledge.

4.2 Output feedback case

In this subsection, we consider the static output feedback controller design for the system (9). Because $x_r(k)$ and $x_d(k)$ are previewable signals, their information can be available in real time similar to the measurable output $\Delta y(k)$ and error signal $e(k)$. To fully utilize all the useful information, the observation equation corresponding to system (9) can be formulated by

$$\bar{y}(k) = \bar{C}\bar{x}(k), \tag{24}$$

where

$$\bar{y}(k) = \begin{bmatrix} e(k) \\ \Delta y(k) \\ x_r(k) \\ x_d(k) \end{bmatrix}, \quad \bar{C} = \begin{bmatrix} I_p & 0 & 0 & 0 \\ 0 & C & 0 & 0 \\ 0 & 0 & I_{p(M_r+1)} & 0 \\ 0 & 0 & 0 & I_{q(M_d+1)} \end{bmatrix}.$$

For the augmented error system (9) and observation equation (24), the following static output feedback controller is considered

$$\Delta u(k) = K_Y \bar{y}(k) = K_Y \bar{C}\bar{x}(k), \tag{25}$$

where K_Y is the gain matrix to be determined. Then substituting this controller into system (9) yields the closed-loop system

$$\bar{x}(k+1) = (\bar{A} + \bar{B}K_Y\bar{C})\bar{x}(k) + F\Delta f_k. \tag{26}$$

The result related to the asymptotic stability analysis of system (26) is summarized in the following theorem, which provides a new synthesis condition within the LMI framework.

Theorem 3: Suppose that Assumptions 1-3 are satisfied. Given scalar $\beta > 0$, matrix Q and invertible matrix W, system (26) is asymptotically stable if there exist matrices $P > 0, L, G_1, G_2, U$ and scalar $\mu > 0$ such that

$$\begin{bmatrix} P - Sym\{G_1\} & * & * & * & * \\ 0 & \mu I_n - Sym\{G_2\} & * & * & * \\ \bar{A}G_1 + \bar{B}LQ & FG_2 & -P & * & * \\ \gamma F_1 G_1 & 0 & 0 & -\mu I_n & * \\ \bar{C}G_1 - UQ & 0 & \beta W^T L^T \bar{B}^T & 0 & -Sym\{\beta UW\} \end{bmatrix} < 0. \tag{27}$$

Furthermore, the gain matrix K_Y is given by $K_Y = LU^{-1}$.

Proof: Consider the following Lyapunov function:

$$V(\bar{x}) = \bar{x}^T P^{-1} \bar{x}.$$

It is clear that V is positive-definite. Computing the difference of $V(\bar{x})$ along with system (26) leads to

$$\begin{aligned} \Delta V\left(\bar{x}(k)\right) &= \bar{x}^T(k)P^{-1}\bar{x}(k) - \bar{x}^T(k-1)P^{-1}\bar{x}(k-1) \\ &= \bar{x}^T(k-1)\left((\bar{A}+\bar{B}K_Y\bar{C})^T P^{-1}(\bar{A}+\bar{B}K_Y\bar{C}) - P^{-1}\right)\bar{x}(k-1) \\ &\quad + 2\bar{x}^T(k-1)(\bar{A}+\bar{B}K_Y\bar{C})^T P^{-1}F\Delta f_{k-1} + \Delta f_{k-1}^T F^T P^{-1} F \Delta f_{k-1}. \end{aligned} \tag{28}$$

Applying (16) and (17) again, one can obtain from (28) that for any constant $\mu > 0$, the following holds:

$$\Delta V\left(\bar{x}(\mathrm{k})\right) \le \bar{x}^T(k-1)\left((\bar{A}+\bar{B}K_Y\bar{C})^T P^{-1}(\bar{A}+\bar{B}K_Y\bar{C}) - P^{-1} + \mu^{-1}\gamma^2 F_1^T F_1\right)\bar{x}(k-1)$$
$$+2\bar{x}^T(k-1)(\bar{A}+\bar{B}K_Y\bar{C})^T P^{-1} F \Delta f_{k-1} + \Delta f_{k-1}^T (F^T P^{-1} F - \mu^{-1} I_n)\Delta f_{k-1}$$
$$= \left[\bar{x}^T(k-1) \quad \Delta f_{k-1}^T\right]\Omega\left[\bar{x}^T(k-1) \quad \Delta f_{k-1}^T\right]^T,$$

where

$$\Omega = \begin{bmatrix} (\bar{A}+\bar{B}K_Y\bar{C})^T P^{-1}(\bar{A}+\bar{B}K_Y\bar{C}) - P^{-1} + \mu^{-1}\gamma^2 F_1^T F_1 & * \\ F^T P^{-1}(\bar{A}+\bar{B}K_Y\bar{C}) & F^T P^{-1} F - \mu^{-1} I_n \end{bmatrix}.$$

Thus, ΔV is negative-definite if $\Omega < 0$. Based on the Lyapunov stability theorem, the closed-loop system (26) is asymptotically stable.

Now we prove that $\Omega < 0$ holds if condition (27) in Theorem 3 is satisfied.

To facilitate the analysis, (27) is rewritten as

$$\left[\begin{array}{c:c} \begin{bmatrix} P - Sym\{G_1\} & * & * & * \\ 0 & \mu I - Sym\{G_2\} & * & * \\ \bar{A}G_1 + \bar{B}LQ & FG_2 & -P & * \\ \gamma F_1 G_1 & 0 & 0 & -\mu I \end{bmatrix} & * \\ \hdashline \beta W^T L^T \bar{B}^T \begin{bmatrix} 0 & 0 & I_s & 0 \end{bmatrix} + UWW^{-1}U^{-1}(\bar{C}G_1 - UQ)\begin{bmatrix} I_s & 0 & 0 & 0 \end{bmatrix} & -Sym\{\beta UW\} \end{array}\right] < 0. \quad (29)$$

Denote

$$T = \begin{bmatrix} P - Sym\{G_1\} & * & * & * \\ 0 & \mu I_n - Sym\{G_2\} & * & * \\ \bar{A}G_1 + \bar{B}LQ & FG_2 & -P & * \\ \gamma F_1 G_1 & 0 & 0 & -\mu I_n \end{bmatrix}, \quad N = UW, \quad M = \begin{bmatrix} 0 \\ 0 \\ I_s \\ 0 \end{bmatrix}\bar{B}LW,$$

$S = W^{-1}U^{-1}(\bar{C}G_1 - UQ)\begin{bmatrix} I_s & 0 & 0 & 0 \end{bmatrix}$. By Lemma 2, (29) ensures

$$\begin{bmatrix} P - Sym\{G_1\} & * & * & * \\ 0 & \mu I_n - Sym\{G_2\} & * & * \\ \bar{A}G_1 + \bar{B}LQ & FG_2 & -P & * \\ \gamma F_1 G_1 & 0 & 0 & -\mu I_n \end{bmatrix} + Sym\left(\begin{bmatrix} 0 \\ 0 \\ I_s \\ 0 \end{bmatrix}\bar{B}LWW^{-1}U^{-1}\left(\bar{C}G_1 - UQ\right)\begin{bmatrix} I_s & 0 & 0 & 0 \end{bmatrix}\right)$$
$$= \begin{bmatrix} P - Sym\{G_1\} & * & * & * \\ 0 & \mu I_n - Sym\{G_2\} & * & * \\ \bar{A}G_1 & FG_2 & -P & * \\ \gamma F_1 G_1 & 0 & 0 & -\mu I_n \end{bmatrix} + Sym\left(\begin{bmatrix} 0 \\ 0 \\ I_s \\ 0 \end{bmatrix}\bar{B}LU^{-1}\left(\bar{C}G_1 - UQ + UQ\right)\begin{bmatrix} I_s & 0 & 0 & 0 \end{bmatrix}\right)$$
$$= \begin{bmatrix} P - Sym\{G_1\} & * & * & * \\ 0 & \mu I_n - Sym\{G_2\} & * & * \\ \bar{A}G_1 & FG_2 & -P & * \\ \gamma F_1 G_1 & 0 & 0 & -\mu I_n \end{bmatrix} + Sym\left(\begin{bmatrix} 0 \\ 0 \\ I_s \\ 0 \end{bmatrix}\bar{B}LU^{-1}\bar{C}G_1\begin{bmatrix} I_s & 0 & 0 & 0 \end{bmatrix}\right)$$
$$< 0.$$

(30)

Denoting $K_Y = LU^{-1}$, the inequality (30) becomes

$$\begin{bmatrix} P-Sym\{G_1\} & * & * & * \\ 0 & \mu I_n - Sym\{G_2\} & * & * \\ \bar{A}G_1 & FG_2 & -P & * \\ \gamma F_1 G_1 & 0 & 0 & -\mu I_n \end{bmatrix} + Sym\left(\begin{bmatrix} 0 \\ 0 \\ I_s \\ 0 \end{bmatrix} \bar{B}K_Y\bar{C}G_1 \begin{bmatrix} I_s & 0 & 0 & 0 \end{bmatrix} \right) < 0,$$

that is,

$$\begin{bmatrix} P-Sym\{G_1\} & * & * & * \\ 0 & \mu I_n - Sym\{G_2\} & * & * \\ \bar{A}G_1 + \bar{B}K_Y\bar{C}G_1 & FG_2 & -P & * \\ \gamma F_1 G_1 & 0 & 0 & -\mu I_n \end{bmatrix} < 0. \tag{31}$$

According to Lemma 3, (31) implies

$$\begin{bmatrix} -G_1^T P^{-1} G_1 & * & * & * \\ 0 & -G_2^T (\mu I_n)^{-1} G_2 & * & * \\ \bar{A}G_1 + \bar{B}K_Y\bar{C}G_1 & FG_2 & -P & * \\ \gamma F_1 G_1 & 0 & 0 & -\mu I_n \end{bmatrix} < 0. \tag{32}$$

Pre- and post-multiplying (32) by the invertible matrix $diag(G_1^{-T}, G_2^{-T}, I_s, I_n)$ and its transpose, respectively, one can obtain that

$$\begin{bmatrix} -P^{-1} & * & * & * \\ 0 & -\mu^{-1} I_n & * & * \\ \bar{A} + \bar{B}K_Y\bar{C} & F & -P & * \\ \gamma F_1 & 0 & 0 & -\mu I_n \end{bmatrix} < 0. \tag{33}$$

Note that $diag(-P, -\mu I_n) < 0$, and applying Lemma 1 to (33) results in $\Omega < 0$. This completes the proof.

Remark 4: Theorem 3 presents a novel static output feedback control design result by reformulating the output equation, which takes full advantage of the available preview information. It should be noted that to calculate the output feedback controller gain K_Y efficiently, β, Q and W are set to be fixed parameters, and then we solve LMI (27), where the variables are P, L, G_1, G_2, U and μ. When LMI (27) has a feasible solution, the controller gain is determined by $K_Y = LU^{-1}$.

Although such a handling method may breed some design conservatism, it allows us to more easily obtain the LMI formulation of the design conditions, which can be directly solved via the MATLAB LMI toolbox [6].

Based on Theorem 3, if LMI (27) is feasible, then the output feedback controller guaranteeing the asymptotic stability of the closed-loop system (26) is given by (25) and $K_Y = LU^{-1}$. To proceed with the design of the tracking controller with preview action, we partition K_Y as

$$K_Y = \begin{bmatrix} K_e & K_y & K_r & K_d \end{bmatrix}, \tag{34}$$

$$K_r = \begin{bmatrix} k_r(0) & k_r(1) & \cdots & k_r(M_r) \end{bmatrix}, \tag{35}$$

$$K_d = \begin{bmatrix} k_d(0) & k_d(1) & \cdots & k_d(M_d) \end{bmatrix}, \tag{36}$$

where $K_e \in R^{m\times p}, K_y \in R^{m\times p}, K_r \in R^{m\times p(M_r+1)}, K_d \in R^{m\times q(M_d+1)}$, $k_r(i) \in R^{m\times p}$ for $i = 0,1,\cdots,M_r$ and $k_d(i) \in R^{m\times q}$ for $i = 0,1,\cdots,M_d$.

Then, (25) can be rewritten as

$$\Delta u(k) = K_e e(k) + K_y \Delta y(k) + \sum_{i=0}^{M_r} k_r(i)\Delta r(k+i) + \sum_{i=0}^{M_d} k_d(i)\Delta d(k+i).$$

From the definition of $\Delta u(k)$, the control input $u(k)$ is computed by

$$\begin{aligned} u(k) &= \sum_{i=0}^{k} \Delta u(i) + u(-1) \\ &= K_e \sum_{i=0}^{k} e(i) + K_y y(k) + \sum_{j=0}^{M_r} k_r(j) r(k+j) + \sum_{j=0}^{M_d} k_d(j) d(k+j) \\ &\quad - \sum_{j=1}^{M_r} k_r(j) r(j-1) - \sum_{j=1}^{M_d} k_d(j) d(j-1), \end{aligned}$$

where it is assumed that $x(i) = 0, u(i) = 0, r(i) = 0$ and $d(i) = 0$ for any $i < 0$.

At this point, we have completed the output feedback preview controller design. The second main result of this paper can be summarized in the following theorem.

Theorem 4: Suppose that Assumptions 1-3 are satisfied. If LMI (27) in Theorem 3 has a feasible solution, then the preview tracking controller of system (1) is

$$
\begin{aligned}
u(k) = K_e \sum_{i=0}^{k} e(i) + K_y y(k) + \sum_{j=0}^{M_r} k_r(j) r(k+j) + \sum_{j=0}^{M_d} k_d(j) d(k+j) \\
- \sum_{j=1}^{M_r} k_r(j) r(j-1) - \sum_{j=1}^{M_d} k_d(j) d(j-1),
\end{aligned}
\tag{37}
$$

where the related feedback gain matrices are given by (34)-(36). Under this controller, the output vector $y(k)$ tracks the reference signal $r(k)$ without static error.

Remark 5: The output feedback-based controller $u(k)$ given by (37) consists of six parts. The second part represents the output feedback control action, and the remaining control parts are the same as in (23). As stressed in Remark 3, such a tracking control scheme also takes full advantage of the available preview information.

Remark 6: Some interesting results have been obtained for Lipschitz nonlinear control systems in [9, 26, 39, 40]. To the best of our knowledge, this paper is the first attempt in the literature to investigate the output tracking control problem with preview action for general Lipschitz systems. The tracking algorithms presented in [26] were developed based on other stringent constraints, such as the requirement that input matrix B and output matrix C are invertible and have at least full column rank, which limits the applications of the proposed methodologies. The adoption of the direct control approach in [20] requires that the input matrix B have special structures. However, in this paper, such restrictions are removed. Moreover, in derivations of controller synthesis conditions (13) and (27), the introduction of matrix variables G_1 and G_2 provides an extra degree of freedom in the solution space for the feasibility problem of matrix inequality. These features indicate that our proposed results are less conservative compared to [20, 26] and are applicable to more general Lipschitz systems. Furthermore, unlike the conventional control scheme [39], our proposed tracking controller includes a novel preview compensation mechanism related to future reference and disturbance in addition to the usual feedback control and integral action, which helps greatly improve the overall output tracking performance of the system.

5 Numerical examples

To validate the performance of our proposed controller, two numerical examples are included in this section.

5.1 State feedback case

We select Chua's circuit to illustrate the applicability of the proposed controller to chaotic physical systems because chaos control has received considerable attention from the scientific community [3, 12, 26]. The dynamics subjected to external disturbance are given by

$$\begin{cases} \dot{x}(t) = Tx(t) + g(x(t)) + Fu(t) + Md(t), \\ y(t) = Cx(t), \end{cases}$$

where

$$T = \begin{bmatrix} -2.548 & 9.1 & 0 \\ 1 & -1 & 1 \\ 0 & -14.2 & 0 \end{bmatrix}, F = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, M = \begin{bmatrix} 0 \\ 0 \\ 10 \end{bmatrix}, C = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix},$$

$$g(x) = \frac{1}{2}\begin{bmatrix} |x_1+1| - |x_1-1| \\ 0 \\ 0 \end{bmatrix}, \quad d(t) = \begin{cases} 1.5, & 0.72 \le t \le 0.96 \\ 0, & others \end{cases}$$

Note that, in this example, the input matrix F has full column rank, thus the right inverse matrix of F does not exist. Additionally, the nonlinear vector function $g(x)$ is not differentiable. Therefore, the control schemes proposed in [26, 39] are not be directly applicable because the basic assumption conditions are not satisfied. In the following solution, the above Chua's circuit model is converted into a discrete-time system through Euler discretization. By applying the theoretical result given in subsection 4.1, a tracking control scheme with preview action can be derived.

When the sampling period is 0.012 s, the approximate discrete-time model is of the following form

$$\begin{cases} x(k+1) = Ax(k) + f(x(k)) + Bu(k) + Dd(k), \\ y(k) = Cx(k), \end{cases}$$

where

$$A=\begin{bmatrix}0.9694 & 0.1092 & 0\\ 0.0120 & 0.9880 & 0.0120\\ 0 & -0.1704 & 1.0000\end{bmatrix}, B=\begin{bmatrix}0\\ 0.0120\\ 0\end{bmatrix}, D=\begin{bmatrix}0\\ 0\\ 0.1200\end{bmatrix}, C=\begin{bmatrix}0 & 0 & 1\end{bmatrix},$$

$$f(x)=0.006\begin{bmatrix}|x_1+1|-|x_1-1|\\ 0\\ 0\end{bmatrix},\quad d(k)=\begin{cases}1.5, & 60\le k\le 80\\ 0, & others\end{cases}$$

By mathematical operations, we obtain

$$\begin{aligned}\|f(x)-f(\bar{x})\| &= 0.006\left|(|x_1+1|-|x_1-1|)-(|\bar{x}_1+1|-|\bar{x}_1-1|)\right|\\ &= 0.006\left|(|x_1+1|-|\bar{x}_1+1|)-(|x_1-1|-|\bar{x}_1-1|)\right|\\ &\le 0.006\left[\left|(|x_1+1|-|\bar{x}_1+1|)\right|+\left|(|x_1-1|-|\bar{x}_1-1|)\right|\right].\end{aligned}$$

Using the absolute value inequality $|(|a|-|b|)|\le|a-b|$, one has

$$\|f(x)-f(\bar{x})\|\le 0.012|x_1-\bar{x}_1|\le 0.012\|x-\bar{x}\|.$$

Thus, $f(x)$ is a globally Lipschitz function and the Lipschitz constant $\gamma=0.012$.

In this example, suppose that the disturbance and the desired reference signals are both previewable. For the purpose of the simulation, the reference signal is selected as

$$r(k)=\begin{cases}0, & k<30\\ 3, & k\ge 30\end{cases} \tag{38}$$

For comparison purposes, three cases are considered separately, including ① $M_r=M_d=0$ (no preview), ② $M_r=2, M_d=1$ and ③ $M_r=7, M_d=3$. By resorting to the MATLAB LMI toolbox, the feasible solutions to LMI (13) in Theorem 1 can be obtained. Furthermore, the controller gains are designed as follows:

① $K_e=55.1327, K_x=\begin{bmatrix}2.3697 & -128.3706 & 271.4128\end{bmatrix}$.

② $K_e=56.2505, K_x=\begin{bmatrix}2.9575 & -129.7889 & 276.9447\end{bmatrix}$,

$K_r=\begin{bmatrix}0.8939 & -53.4206 & -57.5458\end{bmatrix}, K_d=\begin{bmatrix}33.2120 & 26.5790\end{bmatrix}$.

③ $K_e=58.1603, K_x=\begin{bmatrix}2.2160 & -129.5806 & 280.2482\end{bmatrix}$,

$K_r=\begin{bmatrix}0.3658 & -57.0949 & -57.2011 & -49.8302 & -39.2150 & -28.3553 & -19.0604 & -11.8318\end{bmatrix}$,

$K_d = [33.7666 \quad 26.7035 \quad 18.7877 \quad 12.3614]$.

Therefore, according to Theorem 2, the system output under controller (23) can always asymptotically track the desired reference signal despite the nonlinearity and the disturbance. The tracking performance is illustrated by Figs. 2 and 3, from which we can see that the closed-loop transient characteristics, including overshoot and settling time are drastically improved via the preview controller. Additionally, the preview action produces a rapid dynamic response speed for the closed-loop system. Moreover, a preview of the disturbance is shown to be effective in attenuating the fluctuations caused by the disturbance, leading to a better robustness of the system. In addition, by increasing the preview length appropriately, the tracking accuracy can always be enhanced further. The control input is shown in Fig.3, which indicates that the introduction of the preview action does not increase the input amplitude. In summary, the designed preview controller shows satisfactory results and good control performance.

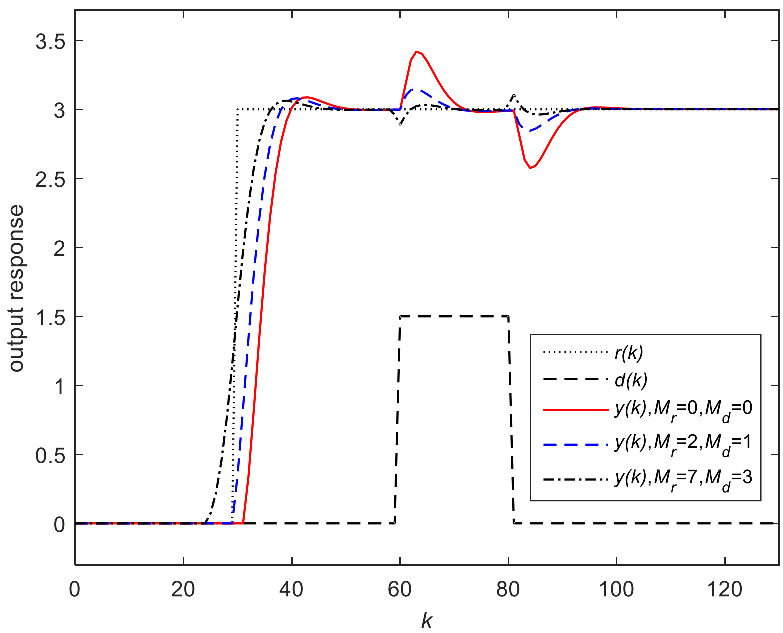

Fig. 1 The output response

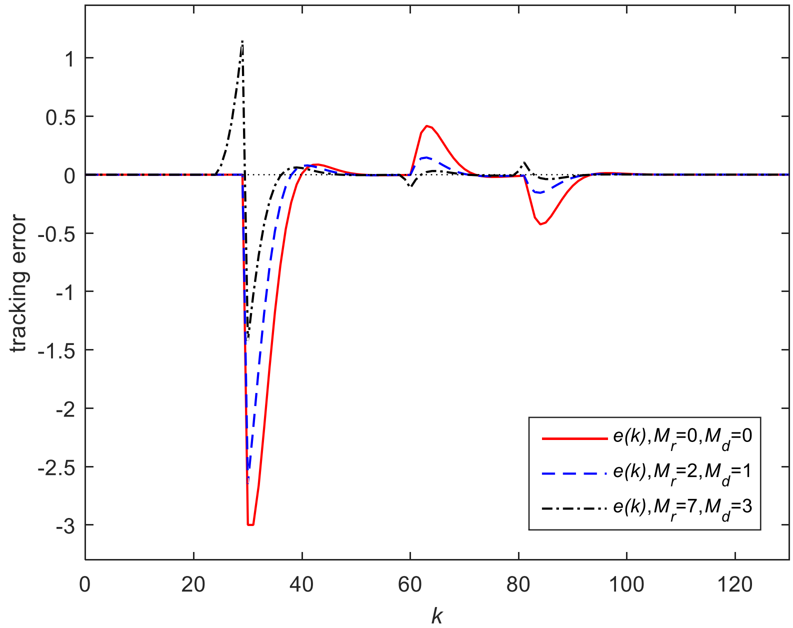

Fig. 2 The tracking error

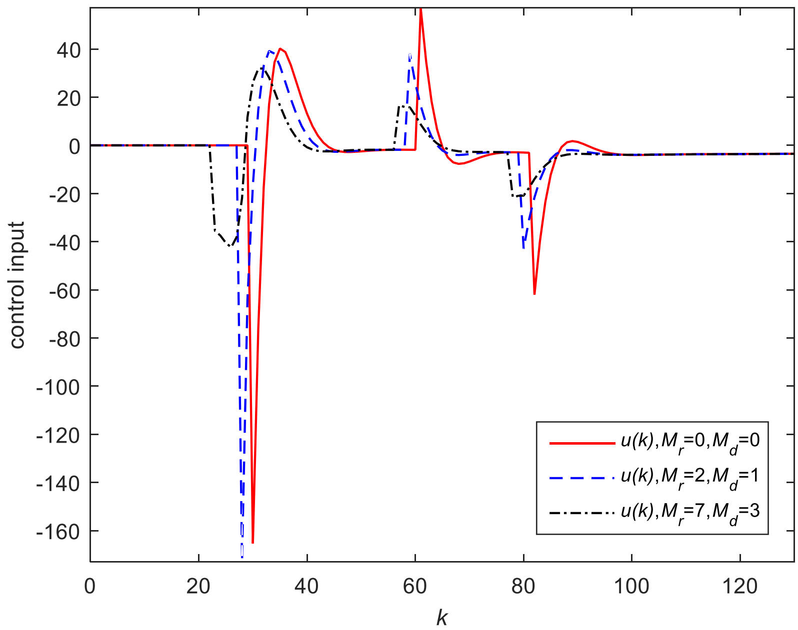

Fig. 3 The control input

Note that the proposed preview controller (23) uses the information on accumulated tracking error from the initial to the current time. This means that the past information continuously affects the current control input. Thus, in the following, we will check the case where abrupt changes of reference and/or disturbance occur more than two times without initializing of control input. For example, the reference signal is changed to

$$r(k)=\begin{cases}0, & k<10\\ 1.5, & 10\le k<50\\ 3, & 50\le k<100\\ 2, & 100\le k<150\\ 1, & k\ge 150\end{cases}$$

and $d(k)\equiv 0$. The tracking control performance is depicted in Fig. 4 with the control signal depicted in Fig. 6. The tracking error is a good tool to evaluate the tracking quality and is shown in Fig. 5. In this situation, we only need to test the effect of the preview action of the reference signal on the tracking performance. In Fig. 4, it can be seen that, despite four abrupt changes occurring at time instant $k=10,\ 50,\ 100,\ 150$, the entire output tracking behavior is acceptable, and the designed controller performs better in terms of rapidity, low overshoot and high tracking precision. These merits mainly benefit from a novel preview compensation mechanism in the proposed tracking control structure.

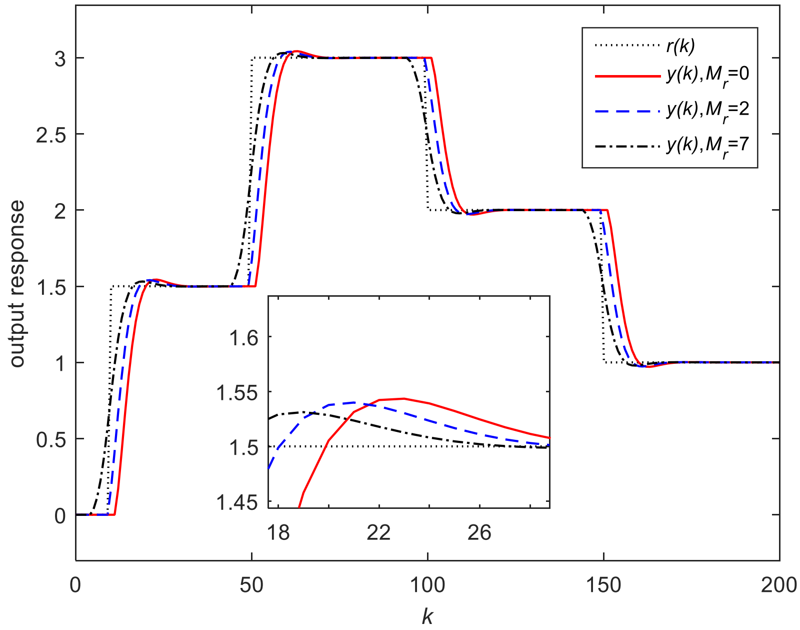

Fig. 4 The output response

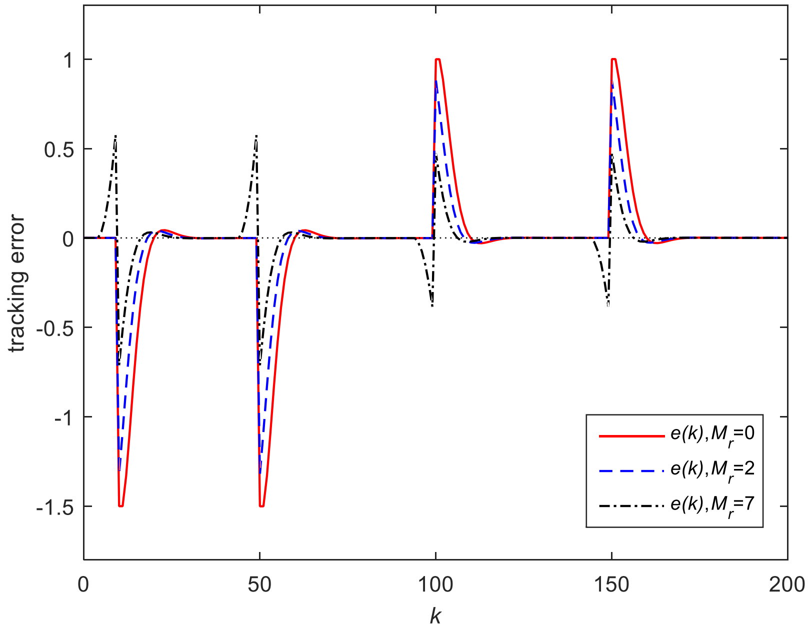

Fig. 5 The tracking error

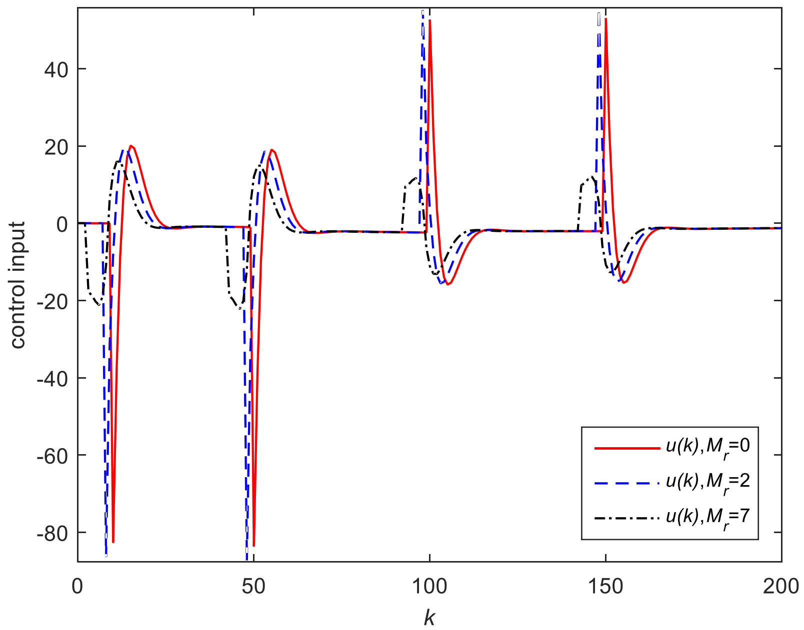

Fig. 6 The control input

5.2 Output feedback case

Consider the nonlinear system (1) described by

$$A=\begin{bmatrix}0.25 & -0.26 & -1.55\\ -0.62 & 0 & 0.35\\ 0.25 & 0.56 & 0\end{bmatrix}, B=\begin{bmatrix}0.65\\ 0\\ 0.67\end{bmatrix}, D=\begin{bmatrix}0\\ 0.17\\ 0.13\end{bmatrix}, C=\begin{bmatrix}-0.55 & 0.25 & 0\end{bmatrix},$$

$$f(x)=\begin{bmatrix} 0.01\cos x_1 \\ 0.015\sqrt{x_2^2+5} \\ 0.02\sin x_3 \end{bmatrix},\quad d(k)=\begin{cases} 0, & k<80 \\ 1.5, & k\geq 80 \end{cases}$$

Clearly, the nonlinearity $f(x)$ is differentiable. According to the mean-value theorem, $f(x)$ is a globally Lipschitz function and the Lipschitz constant $\gamma=0.02$. In this example, both the disturbance and the reference signal are assumed to be previewable.

To compare the effect of the preview length on the tracking performance, three cases are considered, including ① $M_r=0, M_d=0$ (no preview), ② $M_r=2, M_d=1$ and ③ $M_r=8, M_d=3$. Given $\beta=0.8$, $Q=0.65\bar{C}$, $W=\bar{C}\bar{C}^T$. With the help of the MATLAB LMI toolbox, the feasible solutions to LMI (27) in Theorem 3 can be obtained. Furthermore, the controller gains in three situations are designed as follows:

① $K_e=-0.2727, K_y=-0.6935$.

② $K_e=-0.2632, K_y=-0.6966, K_r=\begin{bmatrix}-0.0155 & 0.2341 & 0.2465\end{bmatrix}$,

$K_d=\begin{bmatrix}-0.2627 & -0.1145\end{bmatrix}$.

③ $K_e=-0.2461, K_y=-0.6639$,

$K_r=\begin{bmatrix}-0.0201 & 0.2130 & 0.2341 & 0.2299 & 0.1963 & 0.1597 & 0.1266 & 0.0969 & 0.0701\end{bmatrix}$,

$K_d=\begin{bmatrix}-0.2757 & -0.1175 & -0.0844 & -0.0686\end{bmatrix}$.

Based on Theorem 4, the system output under control law (37) can track the reference signal without static error. In the simulation, (38) is used as the reference signal to demonstrate the tracking performance.

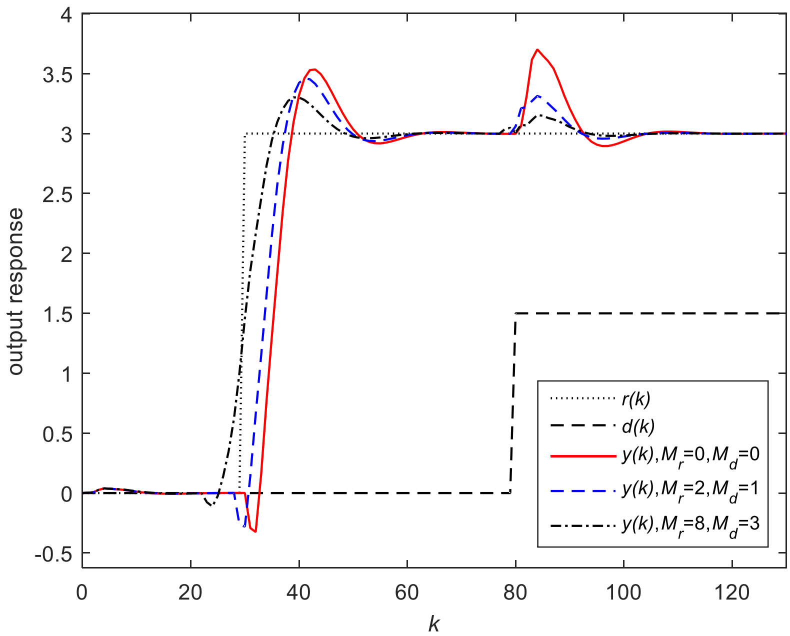

Fig. 7 The output response

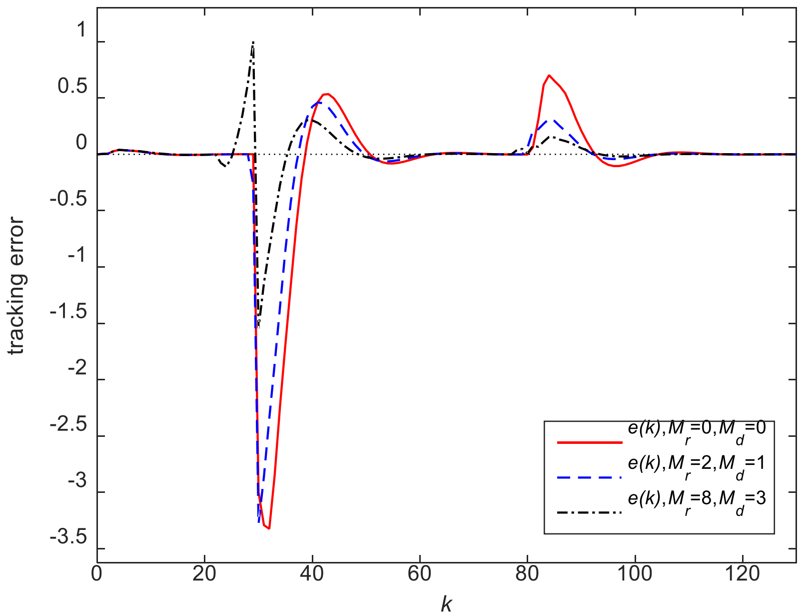

Fig. 8 The tracking error

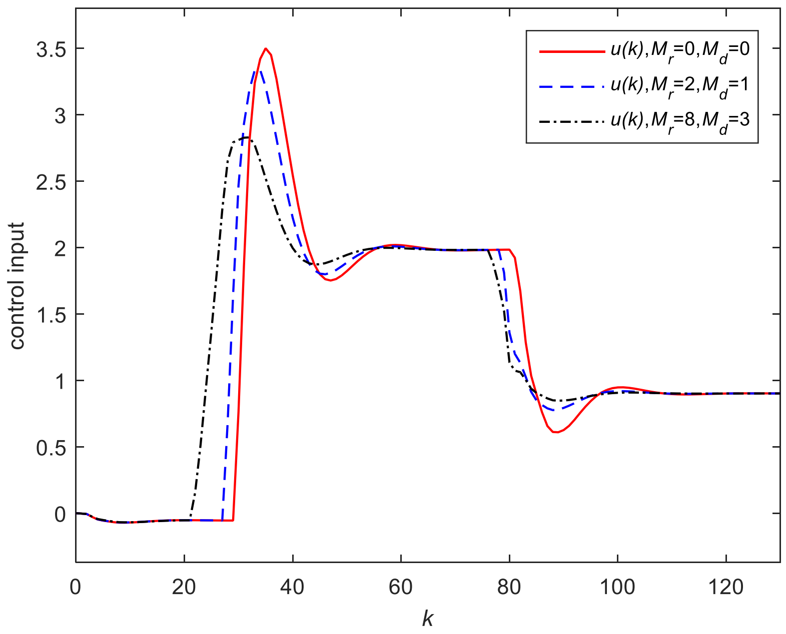

Fig. 9 The control input

Fig. 7 shows the output tracking curves for three cases. Fig. 8 illustrates the corresponding error curves between the practical output and the desired reference. The curves of the control input for different situations are depicted in Fig. 9. As Fig. 7 shows, the system output with the preview controller exhibits better transient behavior compared to the controller with no preview. Specifically, both the overshoot and the settling time are improved. Additionally, when a disturbance signal suddenly occurs at time $k = 80$, the designed preview controller can always perceive and then struggle to compensate for the disturbance. Hence, adding the preview action of disturbances is pretty effective in enhancing the overall control robustness. As shown in Figs. 8 and 9, the preview action makes the closed-loop output track the reference signal more accurately, and the suggested controller with the preview action lowers the input amplitude and produces a better response.

Next, let us consider the following reference signal:

$$r(k) = \begin{cases} 0, & k < 30 \\ 0.15(k-30), & 30 \le k \le 50 \\ 3, & k > 50 \end{cases}$$

The output tracking curve, the tracking error curve and the control input curve are displayed in Figs. 10, 11 and 12, respectively. Apparently, compared to the traditional zero preview controller, fast response, high tracking precision and strong robustness can be achieved by conducting a preview controller. The simulation results confirm the effectiveness and superiority of our proposed preview control method.

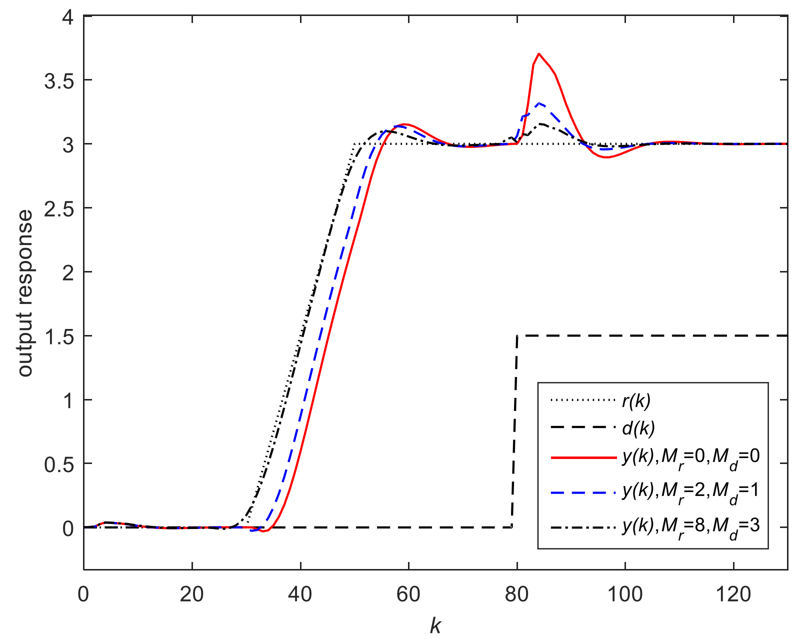

Fig. 10 The output response

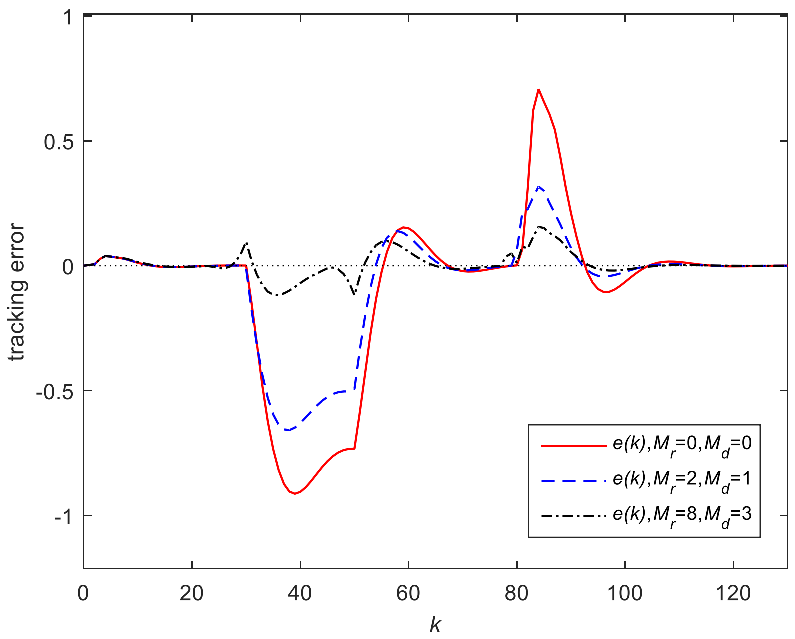

Fig. 11 The tracking error

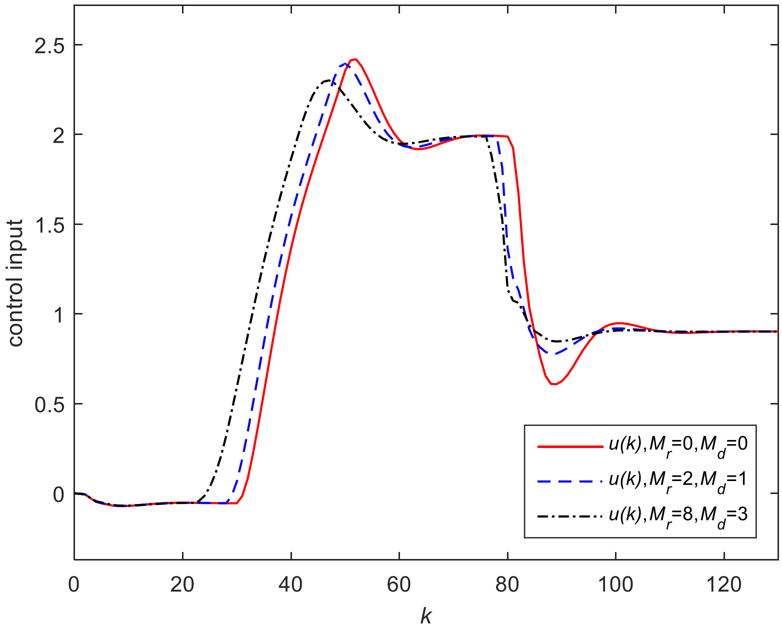

Fig. 12 The control input

6 Conclusions

This paper considers the output tracking controller design for discrete-time Lipschitz nonlinear systems in the framework of a preview control approach. To transform the tracking control problem into a stabilization problem, an augmented error system, which includes preview information and tracking error, is successfully constructed. Then, both the state feedback controller and the static output feedback controller are systematically developed. Additionally, sufficient design conditions for the asymptotic stability of the closed-loop system are established in terms of LMIs based on Lyapunov's second method combined with some special mathematical derivations. Furthermore, two innovative tracking control schemes with preview action for Lipschitz nonlinear systems are obtained, which can drive the system output to the desired reference trajectory. Finally, the effectiveness of the proposed control schemes is validated by numerical simulations.

It should be emphasized that how to further reduce the conservatism of the proposed LMI-based synthesis conditions is a challenging problem. Fortunately, a new formulation of the best less conservative Lipschitz condition, which takes all the properties of the nonlinearities of the system into account, was provided in [44] and was recently applied in [4, 13, 33]. With the aid of this new reformulated Lipschitz condition and the linear parameter varying approach, it is possible to reduce more the conservatism of the obtained results by enlarging the feasible domain of the Lipschitz constant γ, which leaves an interesting research issue for further investigation.

Acknowledgments

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